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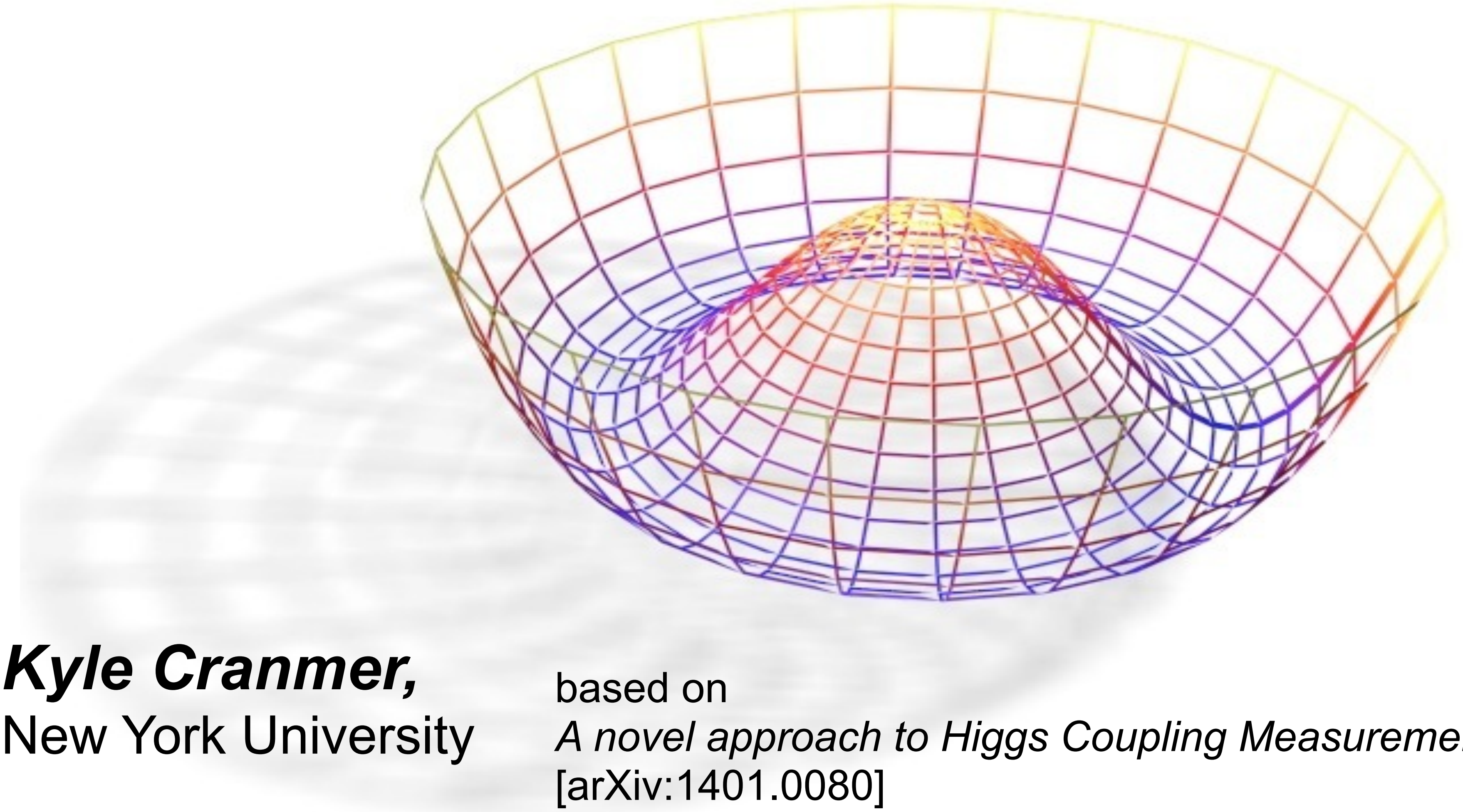
Decoupling Theoretical Uncertainties from Experimental Measurements



Kyle Cranmer,
New York University

based on
A novel approach to Higgs Coupling Measurements
[arXiv:1401.0080]
with Tilman Plehn, Sven Kreiss, & David Lopez-Val

Introduction

Some of the most interesting times in physics come from the comparison of precise theoretical predictions with careful experimental measurements.

While this connection is essential to physics, we generally would like to keep measurements purely observational and free of theoretical input.

- eg, we can relate an observed number of events within some kinematic region to a fiducial cross-section
  - this measurement is independent of precision of theoretical prediction
  - in contrast, to measure a total cross-section we must correct for the efficiency and acceptance in the fiducial region, which relies on theoretical input

This situation is complicated when the physical quantities of interest require multiple measurements with correlated experimental and systematic uncertainties. It is no longer obvious how to decouple theoretical input from the measurement

- this situation is exacerbated by uncertainties in the theoretical inputs
  - with time the theoretical uncertainties may be reduced, so ideally the experimental results would be presented in a way that is still useful at that time

A source of confusion

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It is important to distinguish between the **source** of an uncertainty (parametrized by α) and its **effect** on the expected signal and background $s(\alpha)$ and $b(\alpha)$, which may be a complicated function

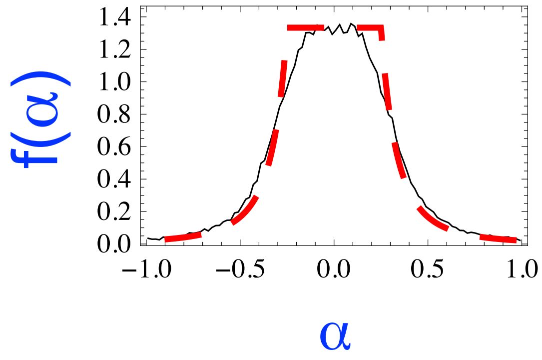
- example:
  - **source** = uncertainty in energy calibration for jets
  - **effect** = change efficiency to pass E_{jet}> 20 GeV cut
- example:

$$\frac{\sigma_{gg}\left(\tau, M_H^2\right)}{\sigma_{gg}^0\left(\tau, M_H^2\right)} = K_{gg}\left(\tau, M_H^2, \alpha_s\right) = 1 + \sum_{n=1}^{\infty} \alpha_s^n(\mu_R) K_{gg}^n\left(\tau, \mu = M_H\right)$$

  - **source** = missing higher order corrections in differential cross section prediction
  - **effect** = change expected number of events and distributions

In addition, we must characterize the magnitude of uncertainty in the source through some prior distribution (“constraint term”) $f(\alpha)$

- but theory uncertainties are not statistical
- choice is controversial



Cacciari & Houdeau [arXiv:1105.5152]
David & Passarino [arXiv:1307.1843]

A simple example

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Imagine we count events in two regions, each populated by two signal processes and some background

$$n_1 = \epsilon_{11}\sigma_1 + \epsilon_{12}\sigma_2 + b_1$$

$$n_2 = \epsilon_{21}\sigma_1 + \epsilon_{22}\sigma_2 + b_2$$

If we are interested in measuring σ_1 & σ_2 we need to know the four selection efficiency factors, which rely on theory

- Equivalently, measure signal strength μ with respect to nominal theoretical reference

$$\epsilon_{12}\sigma_1 \to \mu_1 s_{12}$$

Situation complicated when expected signal and background rates depend on uncertain quantities, parametrized by α

$$n_1 = \mu_1 s_{11}(\alpha) + \mu_2 s_{12}(\alpha) + b_1(\alpha)$$

$$n_2 = \mu_1 s_{21}(\alpha) + \mu_2 s_{22}(\alpha) + b_2(\alpha)$$

An complex example: Higgs @ LHC

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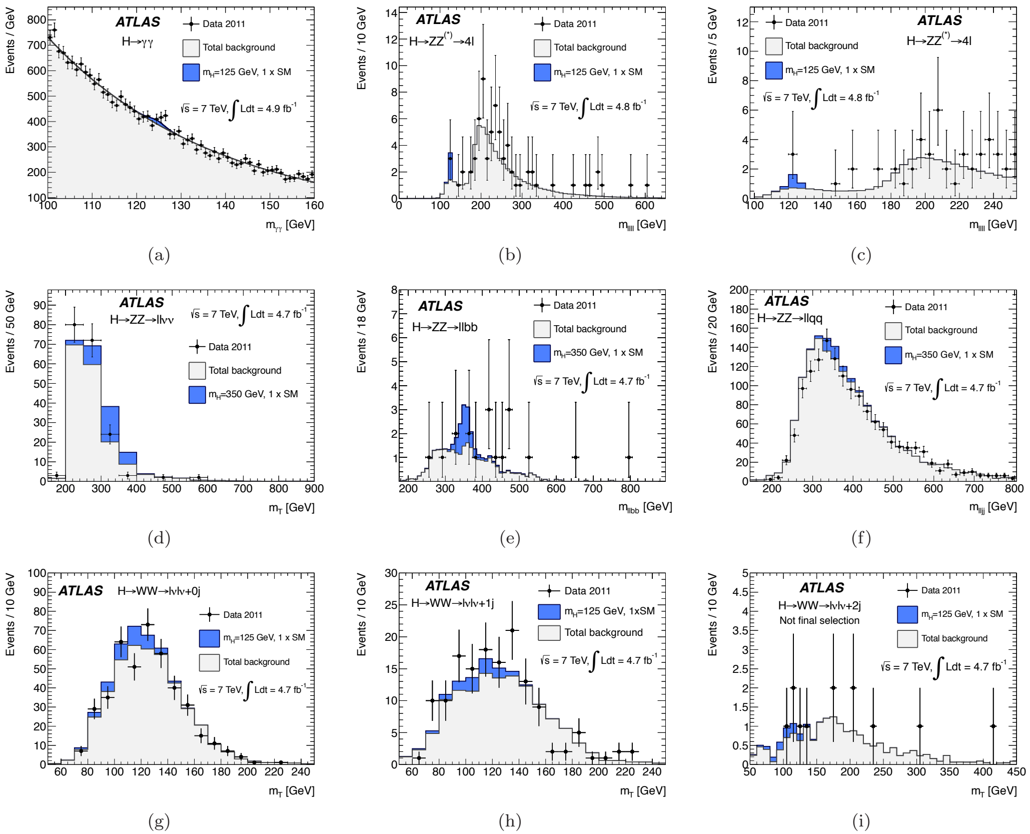

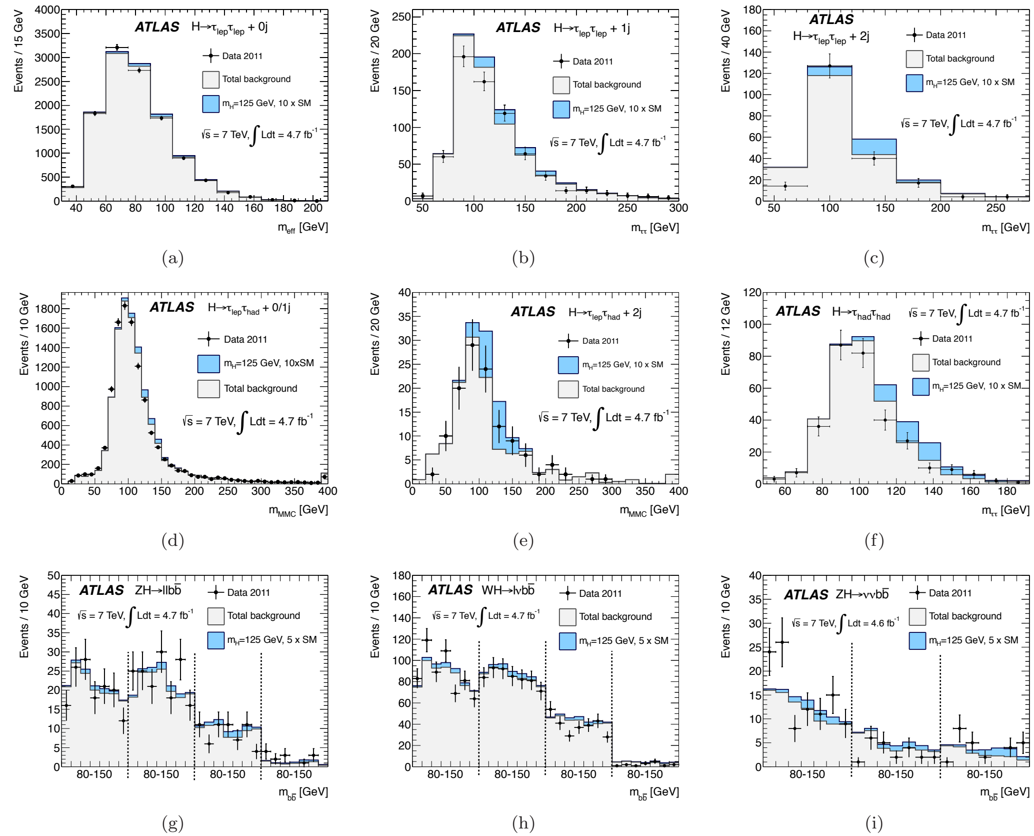

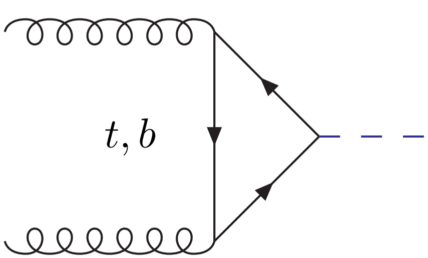

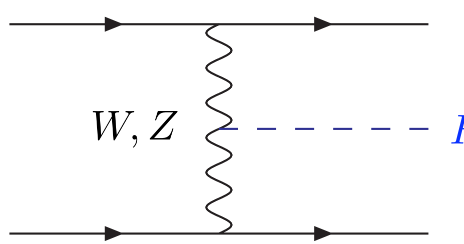

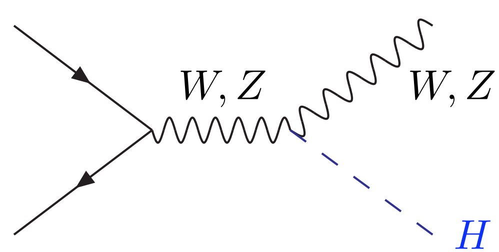

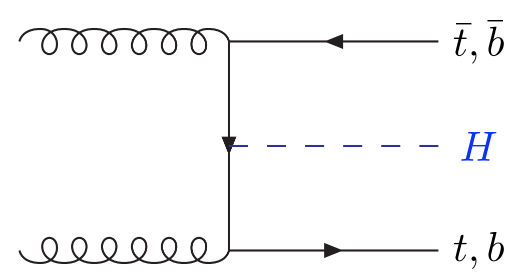

An complex example: Higgs @ LHC

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Channels are sub-divided to enhance sensitivity either for experimental reasons or take advantage of production features

Higgs Boson Decay	Subsequent Decay	Sub-Channels	$\int L\,dt$ [fb^{-1}]	Ref.
		2011 $\sqrt{s}$ =7 TeV		
$H \to ZZ^{(*)}$	4ℓ	$\{4e, 2e2\mu, 2\mu 2e, 4\mu$, 2-jet VBF, ℓ-tag$\}$	4.6	[8]
$H \to \gamma\gamma$	–	10 categories $\{p_{\mathrm{Tt}} \otimes \eta_\gamma \otimes$ conversion$\} \oplus \{$2-jet VBF$\}$	4.8	[7]
$H \to WW^{(*)}$	$\ell\nu\ell\nu$	$\{ee, e\mu, \mu e, \mu\mu\} \otimes \{$0-jet, 1-jet, 2-jet VBF$\}$	4.6	[9]
$H \to \tau\tau$	$\tau_{\mathrm{lep}}\tau_{\mathrm{lep}}$	$\{e\mu\} \otimes \{$0-jet$\} \oplus \{\ell\ell\} \otimes \{$1-jet, 2-jet, $p_{\mathrm{T},\tau\tau} > 100$ GeV, $VH\}$	4.6	[10]
	$\tau_{\mathrm{lep}}\tau_{\mathrm{had}}$	$\{e, \mu\} \otimes \{$0-jet, 1-jet, $p_{\mathrm{T},\tau\tau} > 100$ GeV, 2-jet$\}$	4.6	
	$\tau_{\mathrm{had}}\tau_{\mathrm{had}}$	$\{$1-jet, 2-jet$\}$	4.6	
$VH \to Vbb$	$Z \to \nu\nu$	$E_{\mathrm{T}}^{\mathrm{miss}} \in \{120-160, 160-200, \geq 200$ GeV$\} \otimes \{$2-jet, 3-jet$\}$	4.6	[11]
	$W \to \ell\nu$	$p_{\mathrm{T}}^{W} \in \{< 50, 50-100, 100-150, 150-200, \geq 200$ GeV$\}$	4.7	
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Disentangling multiple production modes

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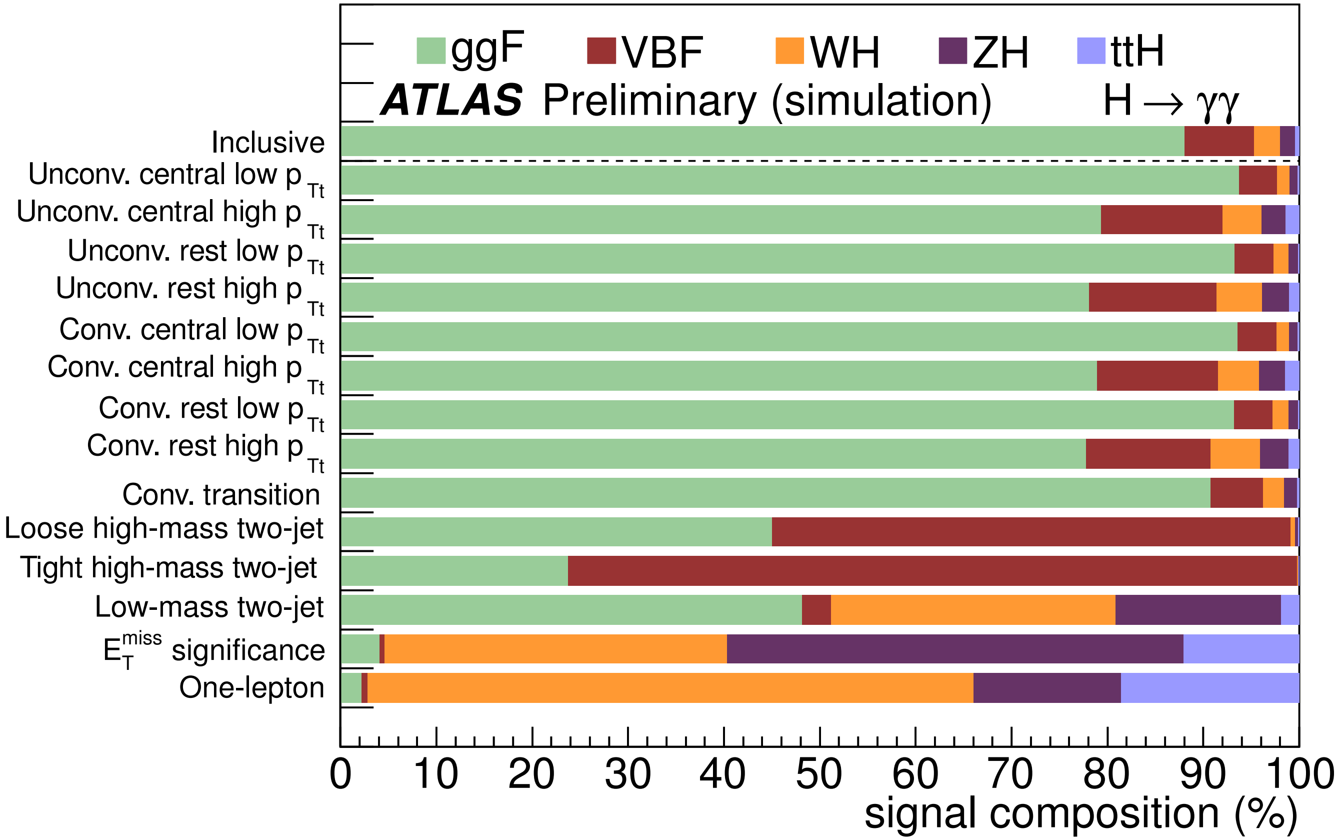
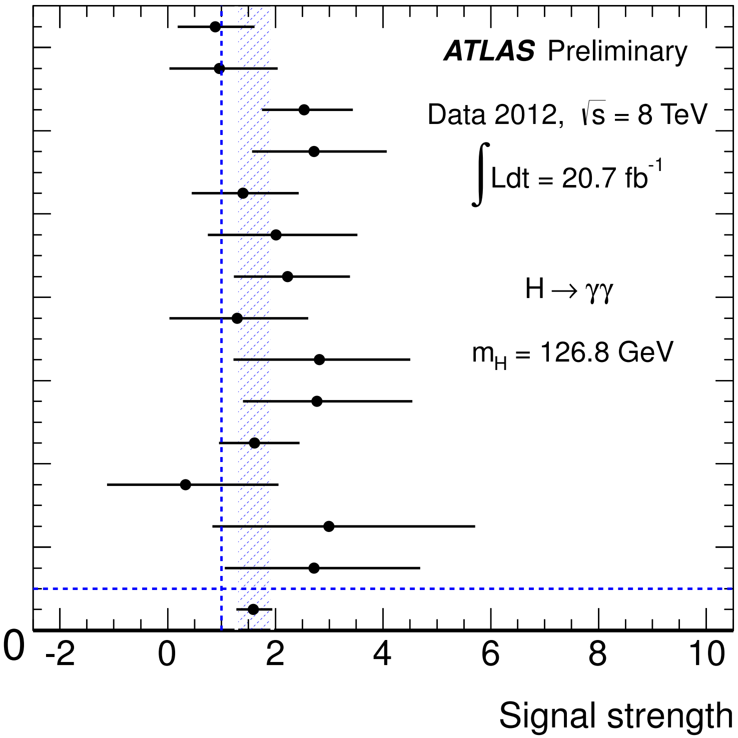




yield shape systematics

$$L_{\rm full}(\boldsymbol{\mu},\boldsymbol{\alpha}) = \underbrace{\prod_{c\in{\rm category}} \left[{\rm Pois}(n_c|\nu_c(\boldsymbol{\mu},\boldsymbol{\alpha})) \prod_{e=1}^{n_c} f_c(x_e|\boldsymbol{\mu},\boldsymbol{\alpha}) \right]}_{\equiv L_{\rm main}(\boldsymbol{\mu},\boldsymbol{\alpha})} \underbrace{\prod_{i\in{\rm syst}} f_i(a_i|\alpha_i)}_{\equiv L_{\rm constr}(\boldsymbol{\alpha})}$$

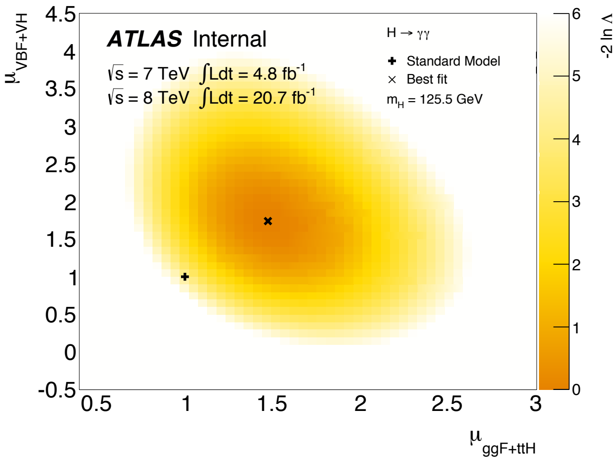
expected number of events: $\nu_c(\boldsymbol{\mu},\boldsymbol{\alpha}) = \sum_{p,d} \mu_{pd}\, s_{cpd}(\boldsymbol{\alpha}) + b_c(\boldsymbol{\alpha})$

Subscripts
- c … category
- p … production
- d … decay
- e … event
- i … systematic

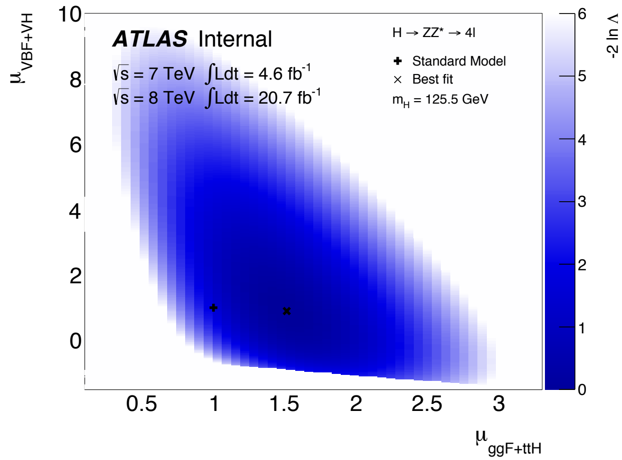
Current presentation of results

Likelihood scans in the space of the “signal strengths” associated to production modes advocated in [arXiv:1307.5865] for communicating LHC Higgs results. Later ATLAS published such scans

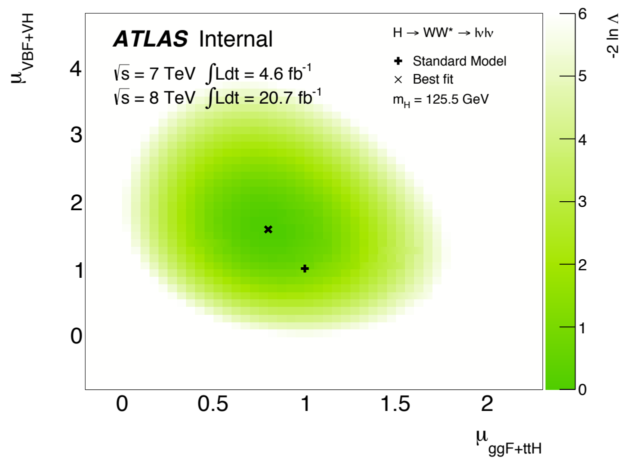
http://doi.org/10.7484/INSPIREHEP.DATA.A78C.HK44



http://doi.org/10.7484/INSPIREHEP.DATA.RF5P.6M3K



http://doi.org/10.7484/INSPIREHEP.DATA.26B4.TY5F



These data are directly linked to the paper in INSPIRE and have been cited:

Information | Citations (4) | Files

Data from Figure 7 from: Measurements of Higgs boson production and couplings in diboson final states with the ATLAS detector at the LHC

ATLAS Collaboration (Aad, Georges (Freiburg U.) [...]) *Show all 2923 authors*

Cite as: ATLAS Collaboration (2013) HepData, http://doi.org/10.7484/INSPIREHEP.DATA.A78C.HK44

Current presentation of results

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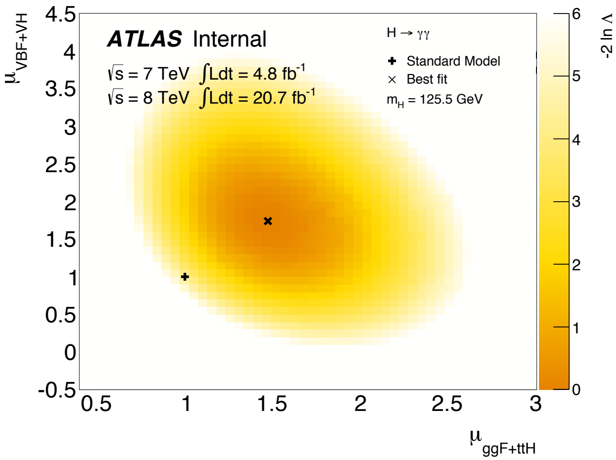
While this is a major step forward in communicating LHC Higgs results, there are some issues that still need to be addressed.

http://doi.org/10.7484/INSPIREHEP.DATA.A78C.HK44



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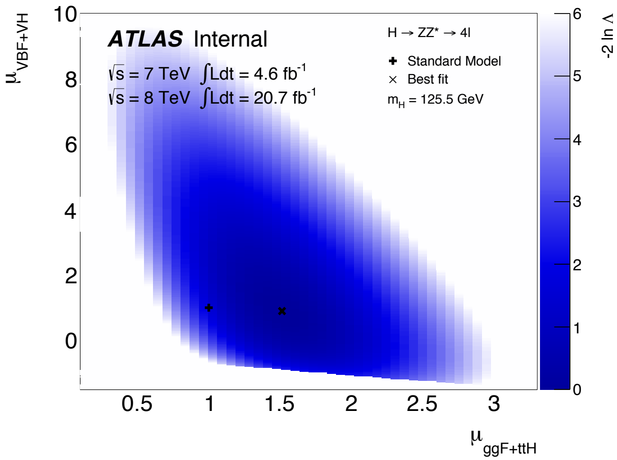

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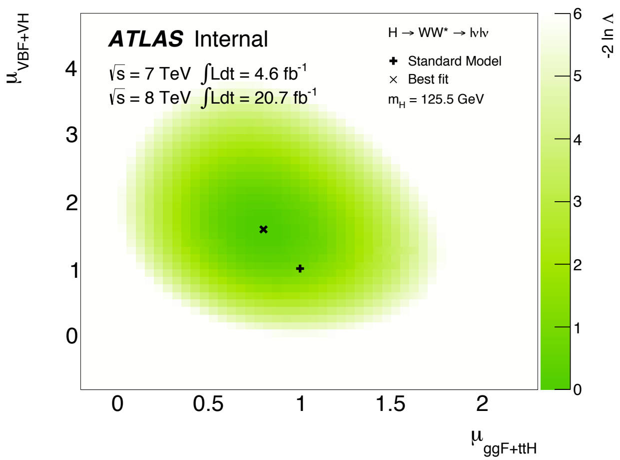

1. Some systematics are shared between these different channels, so simply multiplying them together will lead to **double-counting** those constraint terms (priors)
2. In addition, the **profiling** of the common systematics **is not consistent**, different channels can pull nuisance parameters in different directions
3. **Theory uncertainties** use the standard prescription from the LHC XSWG. That prescription and the magnitude of the uncertainties is **likely to change** in the future as progress is made on the theoretical side.

Goal: We want to **decouple** shared uncertainties from the reported likelihood scan

Basic idea (1/2)

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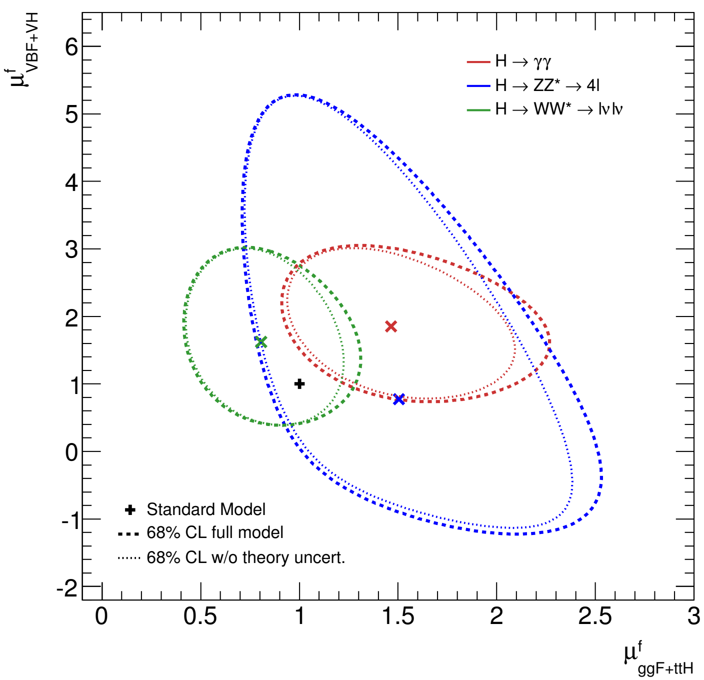
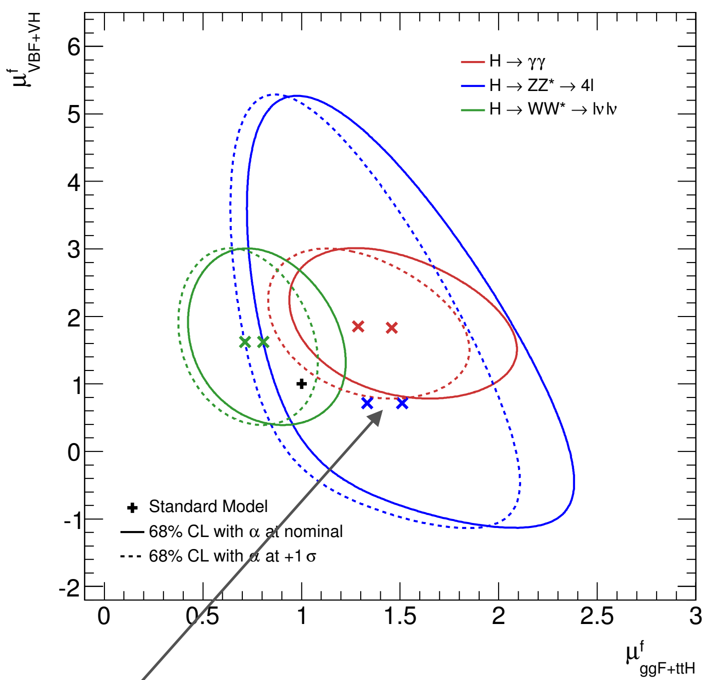
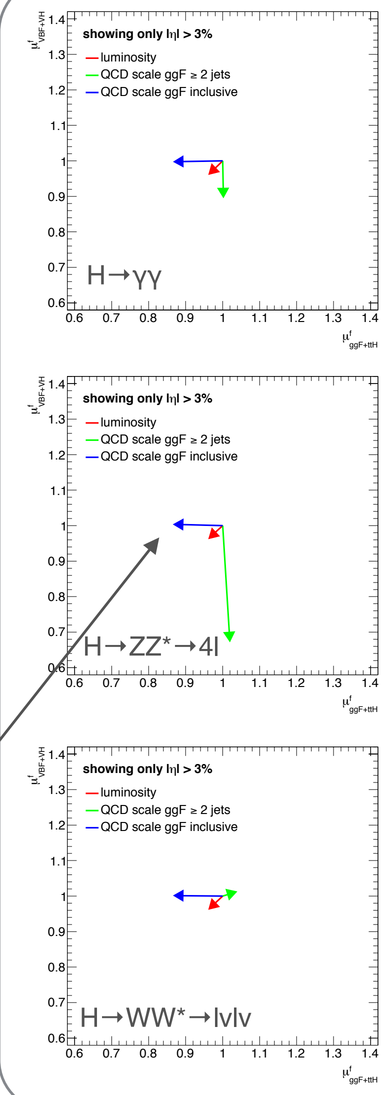
Left: contours with / without theory uncertainties

Center: contours w/o theory uncertainty shifted by changing ggF inclusive x-sec up by 1σ

Right: collection of vectors indicating how best fit point moves due to each source of uncertainty







All plots are based on counting models that mimic ATLAS results.

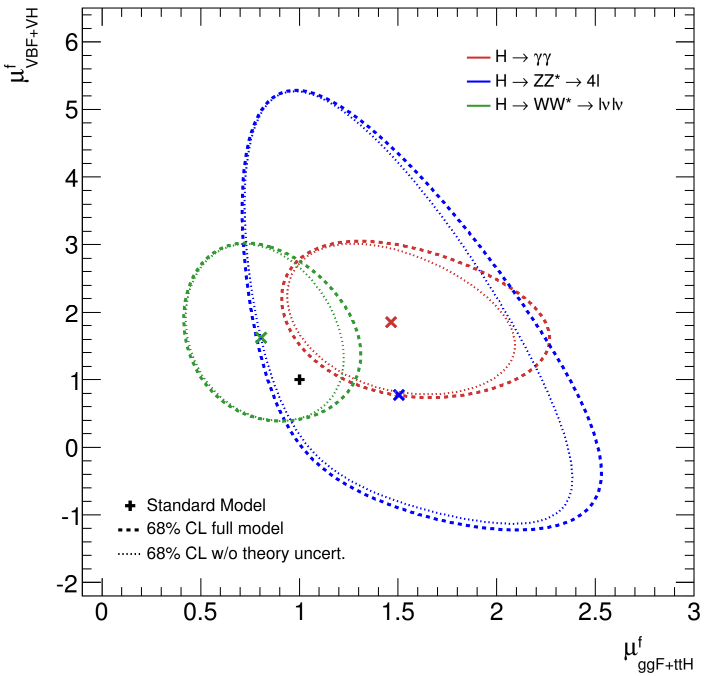
Points move in this plane when varying **common nuisance parameters**.

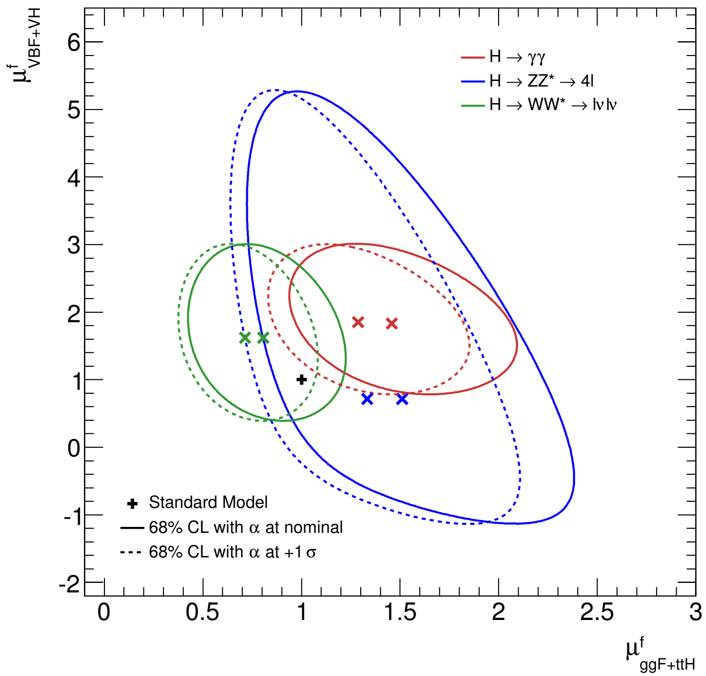
$$\left.\frac{\partial \hat{\mu}_p^{\mathrm{fix}}}{\partial \alpha_i}\right|_{\hat{\boldsymbol{\mu}},\hat{\boldsymbol{\alpha}}} = -\hat{\mu}_p \eta_{ip}$$

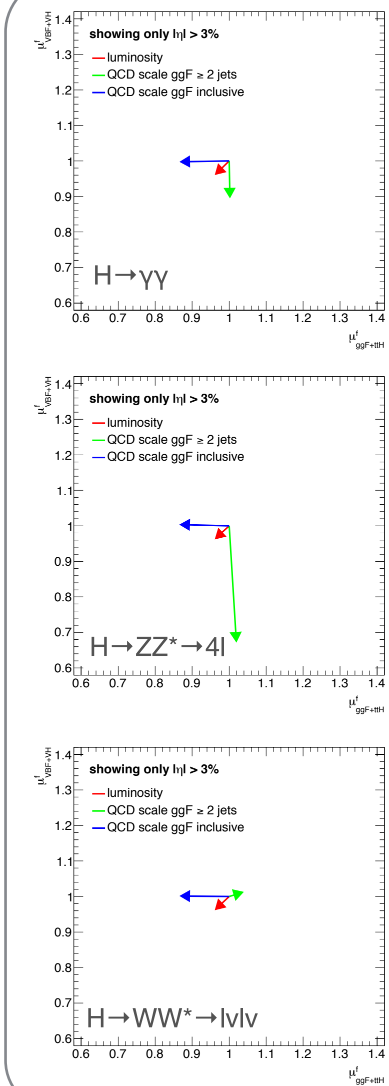
Basic idea (2/2)

Basic idea: Instead of folding the theoretical uncertainties into the experimental result, experiments would publish an **effective likelihood** $L_{\text{eff}}(\mu^{\text{eff}})$ with respect to some fixed theoretical reference and a **reparametrization template** $\mu^{\text{eff}}(\mu,\alpha)$ that documents the affect of individual sources of uncertainty.

theoretical uncertainties are **decoupled** from experimental result!







Then the full likelihood (left) can be **recoupled** by composition

and one is free to modify the constraint term (prior)

$$L_{\text{full}}(\boldsymbol{\mu},\boldsymbol{\alpha}) \approx L_{\text{recouple}}(\boldsymbol{\mu},\boldsymbol{\alpha}) \equiv L_{\text{eff}}(\boldsymbol{\mu}^{\text{eff}}(\boldsymbol{\mu},\boldsymbol{\alpha})) \cdot L_{\text{constr}}(\boldsymbol{\alpha})$$

Effective Likelihoods

Start with the full likelihood function

$$L_{\text{full}}(\boldsymbol{\mu},\boldsymbol{\alpha}) = \underbrace{\prod_{c\in\text{category}}\Big[\overbrace{\text{Pois}(n_c|\nu_c(\boldsymbol{\mu},\boldsymbol{\alpha}))}^{\text{yield}}\prod_{e=1}^{n_c}\overbrace{f_c(x_e|\boldsymbol{\mu},\boldsymbol{\alpha})}^{\text{shape}}\Big]}_{\equiv L_{\text{main}}(\boldsymbol{\mu},\boldsymbol{\alpha})}\underbrace{\overbrace{\prod_{i\in\text{syst}} f_i(a_i|\alpha_i)}^{\text{systematics}}}_{\equiv L_{\text{constr}}(\boldsymbol{\alpha})}$$

Subscripts
c … category
p … production
d … decay

Expected number of events ν has signal strength μ that scales a signal yield s which depends on $\boldsymbol{\alpha}$

$$\nu_c(\boldsymbol{\mu},\boldsymbol{\alpha}) = \sum_{p,d}\mu_{pd}\, s_{cpd}(\boldsymbol{\alpha}) + b_c(\boldsymbol{\alpha})$$

Introduce μ^{eff} that scales with respect to some fixed theoretical reference at $\boldsymbol{\alpha}_0$.
Absorb $\boldsymbol{\alpha}$-dependence into $\mu^{\text{eff}}(\mu,\alpha)$

$$\nu_c(\boldsymbol{\mu},\boldsymbol{\alpha}) \to \sum_{p,d}\mu^{\text{eff}}_{cpd}(\boldsymbol{\mu},\boldsymbol{\alpha})\; s_{cpd}(\boldsymbol{\alpha}_0) + b_c(\boldsymbol{\alpha}_0)$$

Here $\mu^{\text{eff}}(\mu,\alpha)$ is a function that absorbs the dependence on α, but we can also think of μ^{eff} as a parameter on its own and measure $L_{\text{eff}}(\mu^{\text{eff}})$

$$L_{\text{full}}(\boldsymbol{\mu},\boldsymbol{\alpha}) \approx L_{\text{recouple}}(\boldsymbol{\mu},\boldsymbol{\alpha}) \equiv L_{\text{eff}}(\boldsymbol{\mu}^{\text{eff}}(\boldsymbol{\mu},\boldsymbol{\alpha}))\cdot L_{\text{constr}}(\boldsymbol{\alpha})$$

Need a **reparametrization template** $\mu^{\text{eff}}(\mu,\alpha)$ with parameters η and a **method** to determine the ηs such that L_{recouple} approximates L_{full}:

Reparametrization Templates

For uncertainties that affect the **signal** yield inclusively

$$s_{cpd}(\boldsymbol{\alpha}) = s_{cpd}(\boldsymbol{\alpha}_0)\left[1 + \sum_i \eta_{pi}(\alpha_i - \alpha_{0,i})\right] \longrightarrow \mu_{pd}^{\rm eff}(\boldsymbol{\mu}, \boldsymbol{\alpha}) = \mu_{pd}\left[1 + \sum_i \eta_{pi}(\alpha_i - \alpha_{0,i})\right]$$

as in the full likelihood, bi-linear in (μ,α)

Similarly, for nuisance parameters that affect **background** rates:

$$b_c(\boldsymbol{\alpha}) = b_c(\boldsymbol{\alpha}_0)\left[1 + \sum_i \phi_{ci}(\alpha_i - \alpha_{0,i})\right] \longrightarrow \mu_{pd}^{\rm eff}(\boldsymbol{\mu}, \boldsymbol{\alpha}) = \mu_{pd} + \frac{b_c(\boldsymbol{\alpha}_0)}{s_{cpd}(\boldsymbol{\alpha}_0)}\left[\sum_i \phi_{ci}(\alpha_i - \alpha_{0,i})\right]$$

as in the full likelihood, linear in μ and α

For “**cross-talk**” nuisance parameters:

Example: a ggF+2j uncertainty affecting
VBF signal yield through ggF contamination
in a VBF-optimized category: p=VBF, p’=ggF

$$\mu_{pd}^{\rm eff}(\boldsymbol{\mu}, \boldsymbol{\alpha}) = \mu_{pd} + \sum_{i,p'} \mu_{p'd}\, \eta_{pi}^{p'}\,(\alpha_i - \alpha_{0,i})$$

A flexible reparametrization template including above effects

$$\mu_{pd}^{\rm eff}(\boldsymbol{\mu}, \boldsymbol{\alpha}) = \mu_{pd} + \sum_{i,p'} \mu_{p'd}\, \eta_{pi}^{p'}\,(\alpha_i - \alpha_{0,i}) + \sum_i \phi_i(\alpha_i - \alpha_{0,i})$$

Paper outlines method needed to determine and coefficients of template

A toy example with large uncertainties

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Three examples for a simple 2 channel case with large uncertainties.

The recoupled likelihood excellent approximation to full likelihood

Contour with no theory uncertainty much smaller

Contour with theory uncertainty much larger. Here treated using Gaussian constraint term (prior)

Modified contour switching from Gaussian to RFIT / box constraint.

Anyone can do in recoupled approach, but would require a experiments to re-run in current approach.

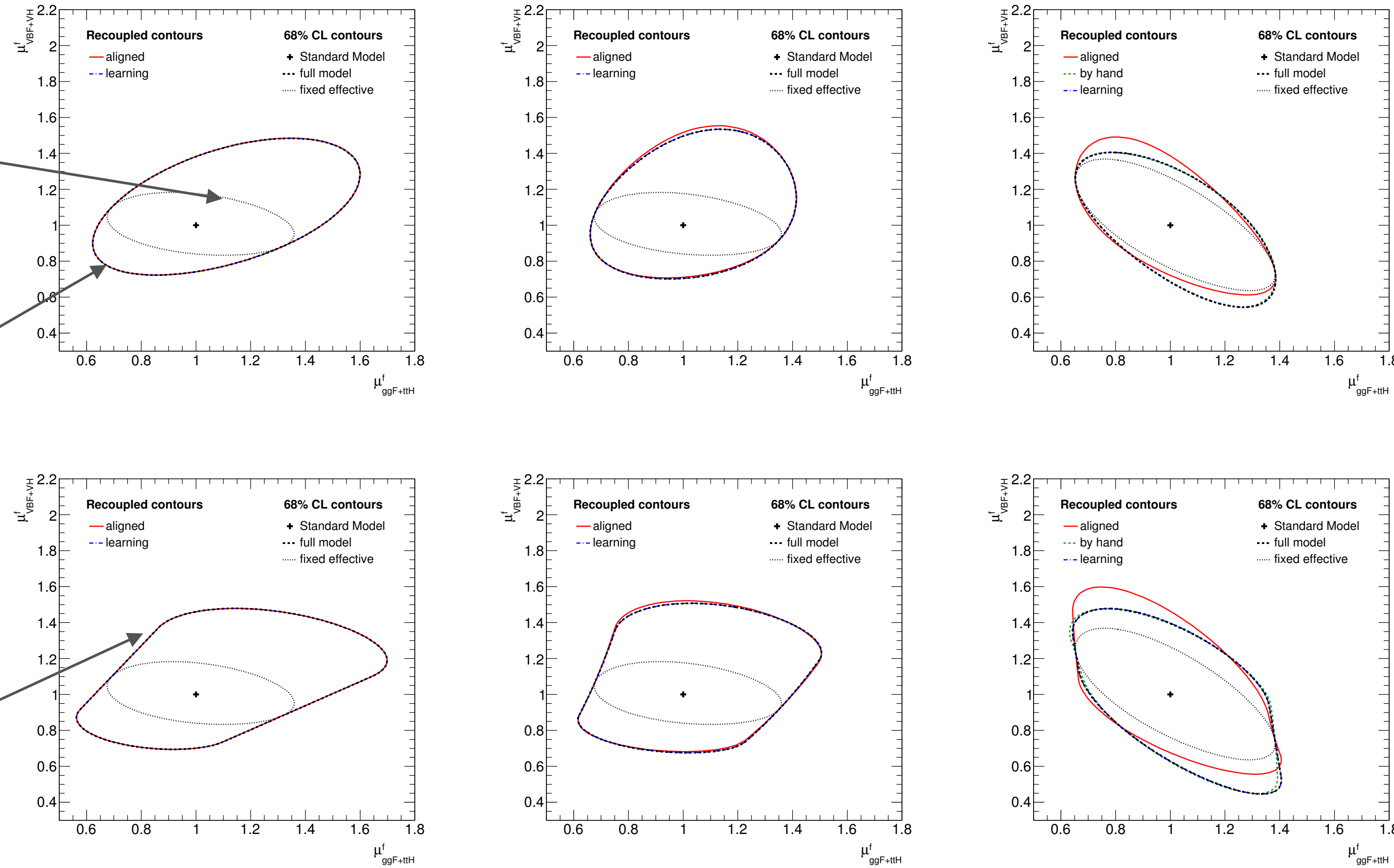

Figure 7. Comparison of full likelihood (solid) and recouped (dashed) likelihood for Scenarios A, B, and C. Scenario C illustrates the impact of using three templates 'aligned' (red), 'by hand' (green), and 'learning' (blue) as described in the text. The top row is based on the nominal Gaussian constraint and the bottom row shows the result of replacing it with an alternative RFIT constraint term. The effective likelihood with $\alpha = 0$ is shown as a dotted line.

Results with combined Higgs benchmark

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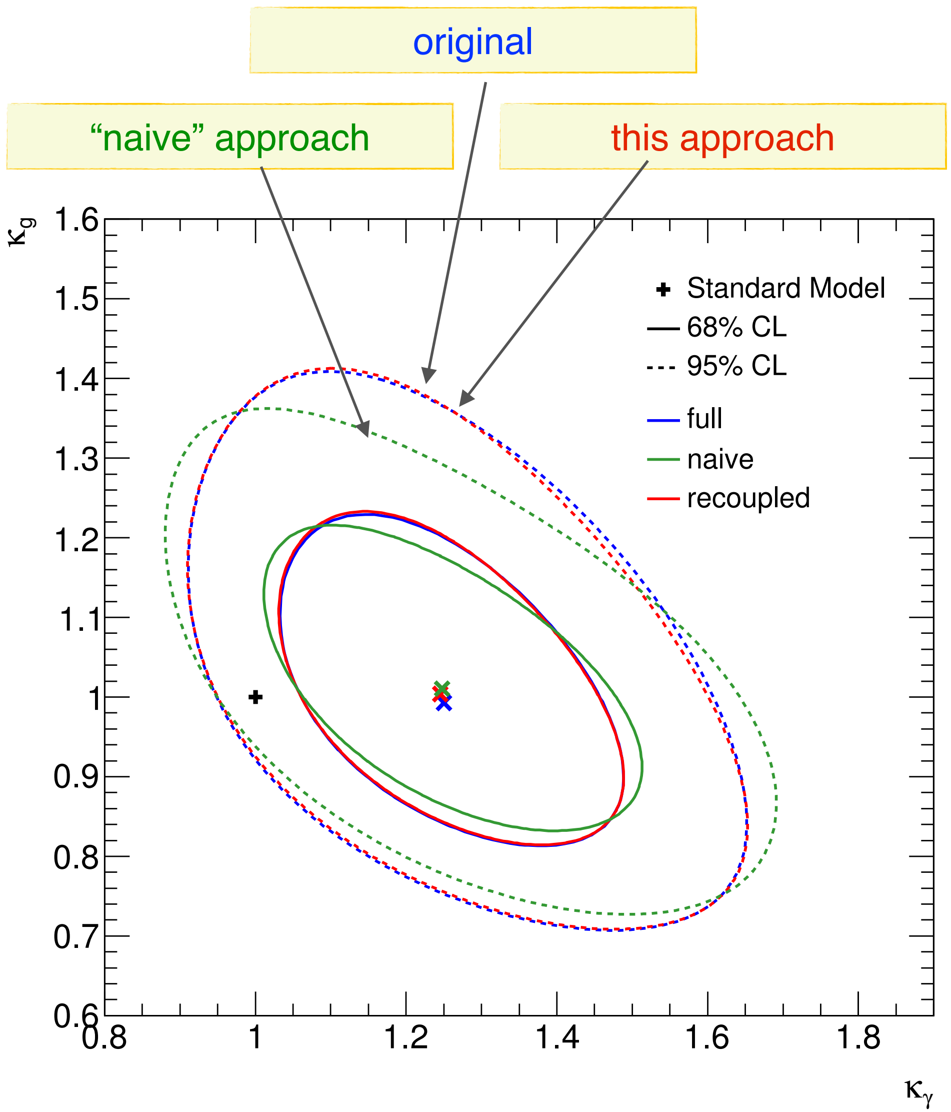
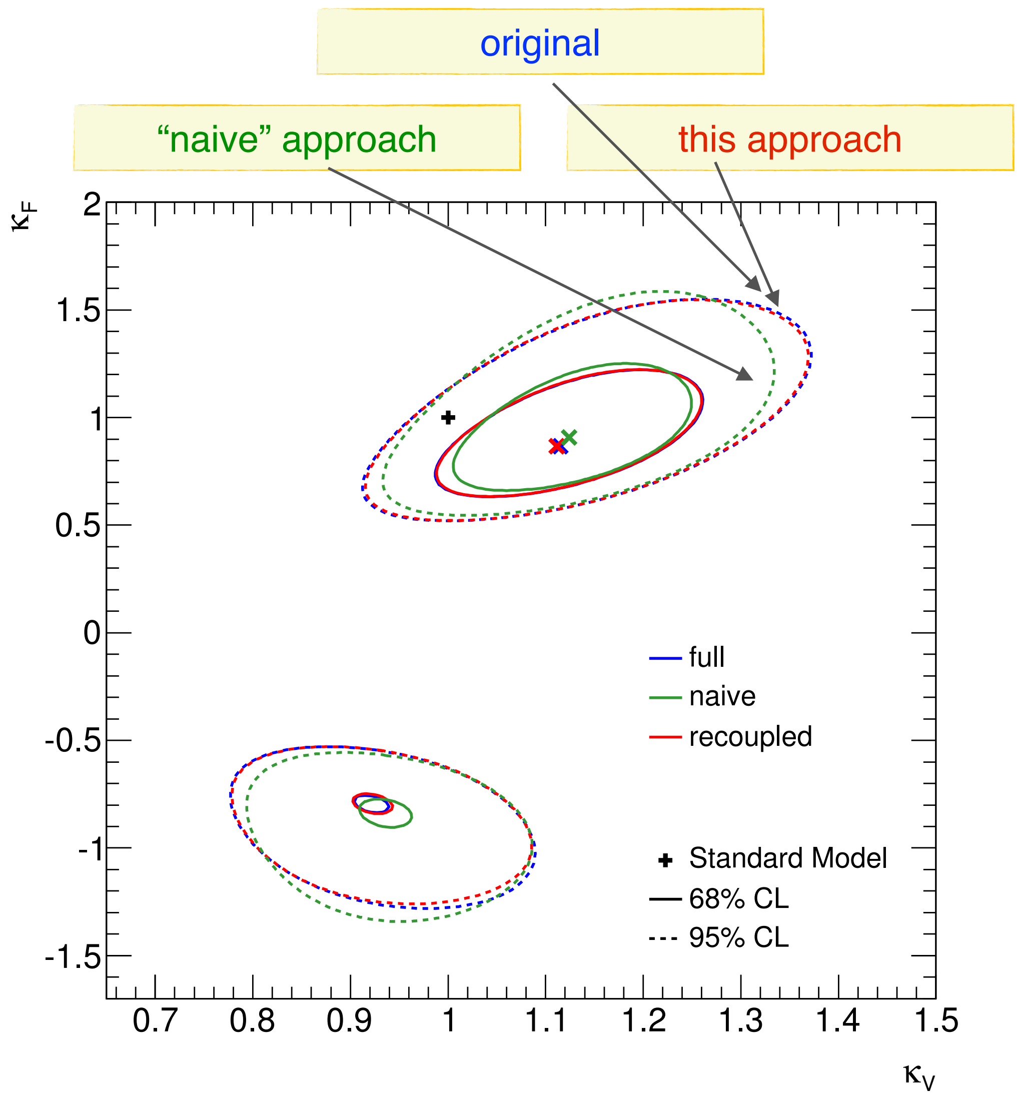
Here are results for two Higgs coupling benchmark models





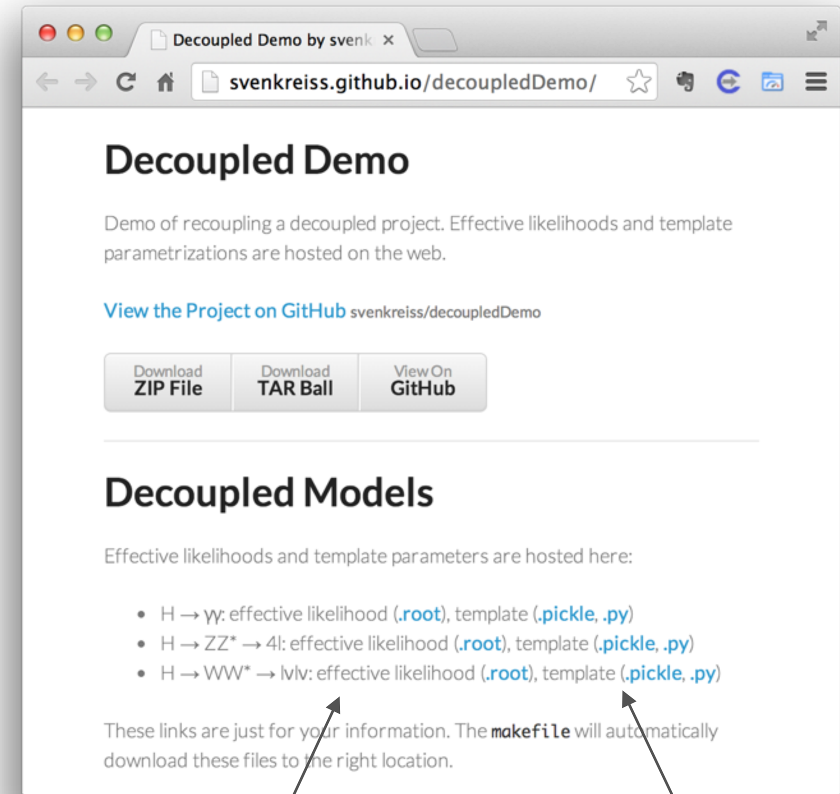
All plots are based on counting models that mimic ATLAS results.

Combination of Effective Likelihoods

Center for Cosmology and Particle Physics

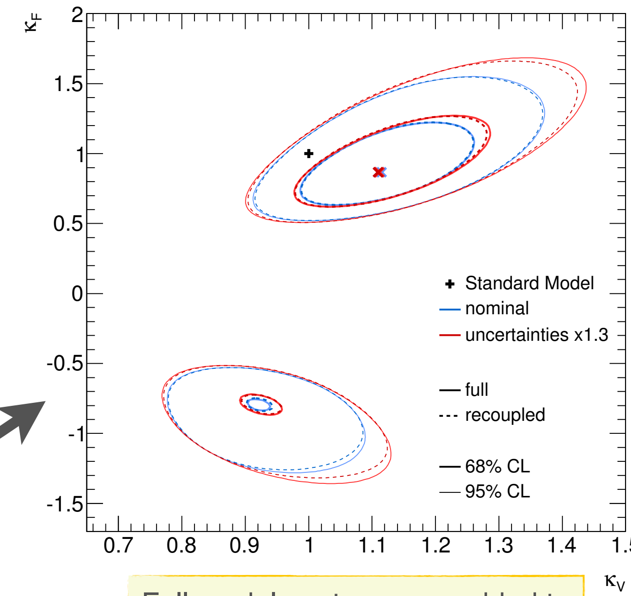
Demo at https://github.com/svenkreiss/decoupledDemo (works on lxplus).

L_{eff} is an efficient lookup table that replaces most of the complexity of the full model. What is normally a job for a cluster runs in 16min on my laptop.

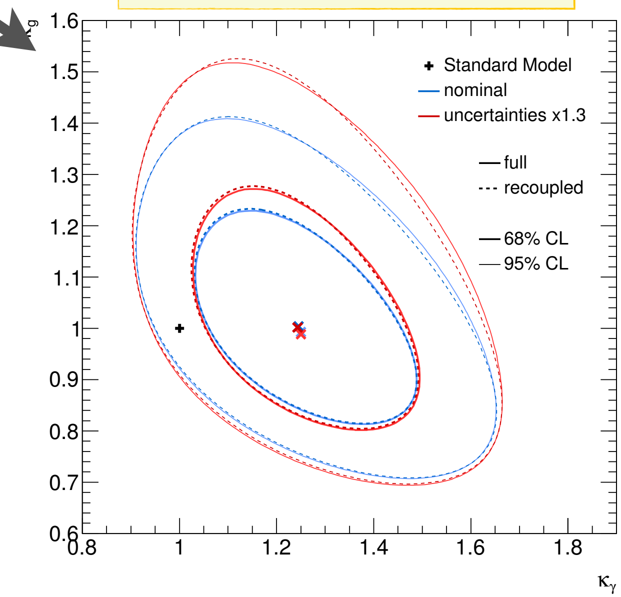


Effective likelihoods and **template** to common nuisance parameters can be published on the web.

Recoupling with two prescriptions for theory uncertainties.



Full model contours are added to the plots for comparison



Conclusions

The ability to decouple theoretical uncertainties from experimental results would be a big step forward

- theory uncertainties are **not statistical** in nature and will evolve with time
- this technique gives a lot of **flexibility** in how they are handled

We have outlined a technique that achieves this

- In addition, the technique solves a problem associated to double counting constraint terms and inconsistent profiling that is present if we publish profile likelihood scans for the individual channels.

This approach still requires that the experiments understand the effect of individual sources of uncertainty on the various channels in the way that we are doing it now coordinated via the LHC HXSWG.

Thank you!

Determining Template Parameters

The covariance matrix can be used to determine up to $n_p \cdot n_\alpha$ of the template parameters η. For example, for templates without "cross-talk" and only category universal, symmetric uncertainties, the template parameters η are determined by:

$$\left.\frac{\partial \hat{\mu}_p^{\mathrm{fix}}}{\partial \alpha_i}\right|_{\hat{\boldsymbol{\mu}},\hat{\boldsymbol{\alpha}}} = -\hat{\mu}_p \eta_{ip}$$

Derivation is in the paper with worked examples in the appendix.

Similar equations can be derived for other scenarios, but usually knowledge has to be added "by-hand" to keep the number of parameters $\leq n_p \cdot n_\alpha$.

For more general templates, the local information contained in the likelihood and its first and second derivative is not enough, and information from various points of the likelihood needs to be used. This can be done by minimizing a loss function with the full and recoupled likelihoods:

$$\mathrm{Loss}(\boldsymbol{\eta}) = \int d\boldsymbol{\mu} d\boldsymbol{\alpha}\, \pi(\boldsymbol{\mu}, \boldsymbol{\alpha})\ \left| L_{\mathrm{full}}(\boldsymbol{\mu}, \boldsymbol{\alpha}) - L_{\mathrm{recouple}}(\boldsymbol{\mu}, \boldsymbol{\alpha}; \boldsymbol{\eta}) \right|^2$$

where π(μ,α) is a weight function. One possibility is to treat π(μ,α) as a posterior obtained using a baseline constraint term: $\pi(\boldsymbol{\mu}, \boldsymbol{\alpha}) \propto L_{\mathrm{main}}(\boldsymbol{\mu}, \boldsymbol{\alpha}) L_{\mathrm{constr}}(\boldsymbol{\alpha})$

In practical terms, this means that the integral in the loss function can be obtained using MCMC [note, does not make the method Bayesian].

BSM Models in Prod Mode Plan

So far, we publish a SM point in the 2-d plane. However, it is also possible to come up with a 1-parameter description of various BSM models and show them as a line in the same plane.

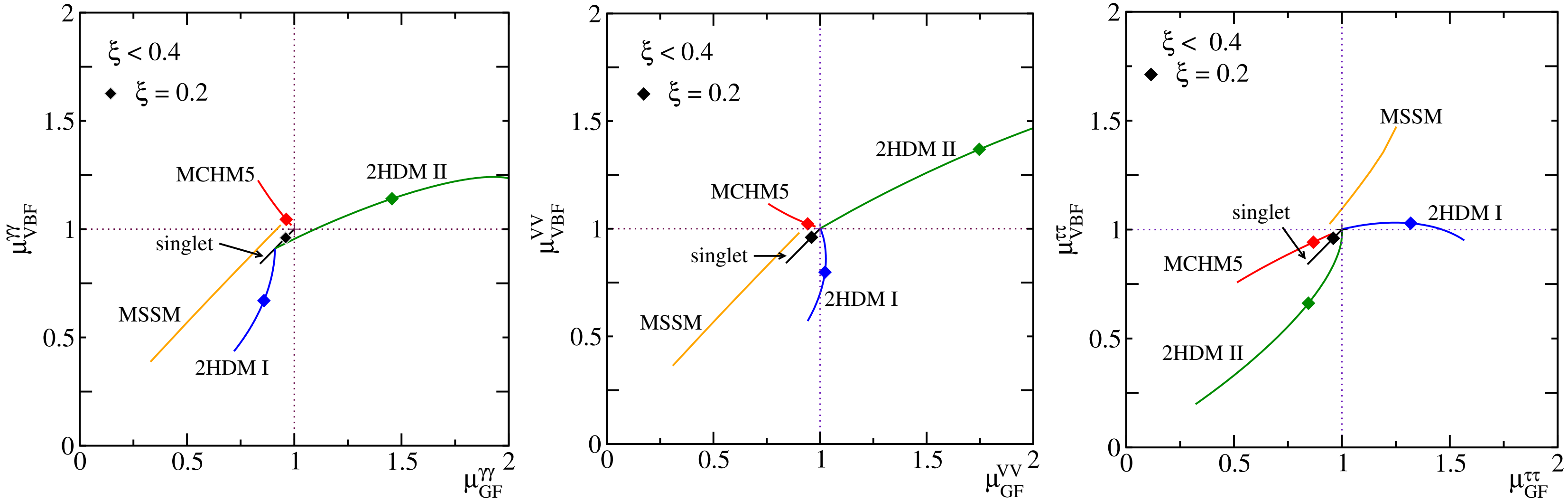


Figure 5. Decay–diagonal correlations of signal strengths $\mu_{\mathrm{GF},d}$ vs $\mu_{\mathrm{VBF},d}$ for $d = \gamma\gamma, VV, \tau\tau$ in different models. The coupling variation is limited to $\xi < 0.4$ and the value $\xi = 0.2$ is singled out. The slight deviations from a complete decoupling are discussed in the text.

Robustness R in the three channels of various BSM models to changes in the QCD scale for the inclusive and ≥2 jets bins.

$$R_i(\boldsymbol{\mu}) = \frac{|\boldsymbol{\mu} - \mathbf{1}|^2 \, |\partial_{\alpha_i} \boldsymbol{\mu}^{\text{fix}}|}{(\boldsymbol{\mu} - \mathbf{1}) \cdot (\partial_{\alpha_i} \boldsymbol{\mu}^{\text{fix}})}$$

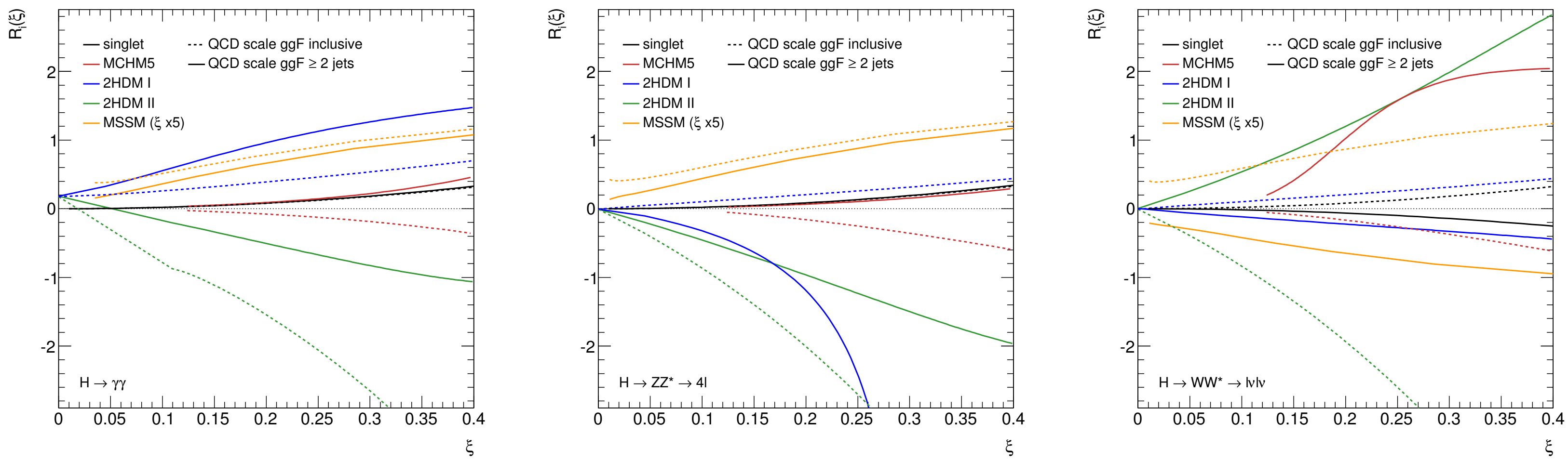


Figure 6. The sensitivity heuristic $R_i(\xi)$ evaluated for various new physics models and the theoretical uncertainties i associated to the gluon fusion cross section for $\geq$ 0-jets and $\geq$ 2-jets.